

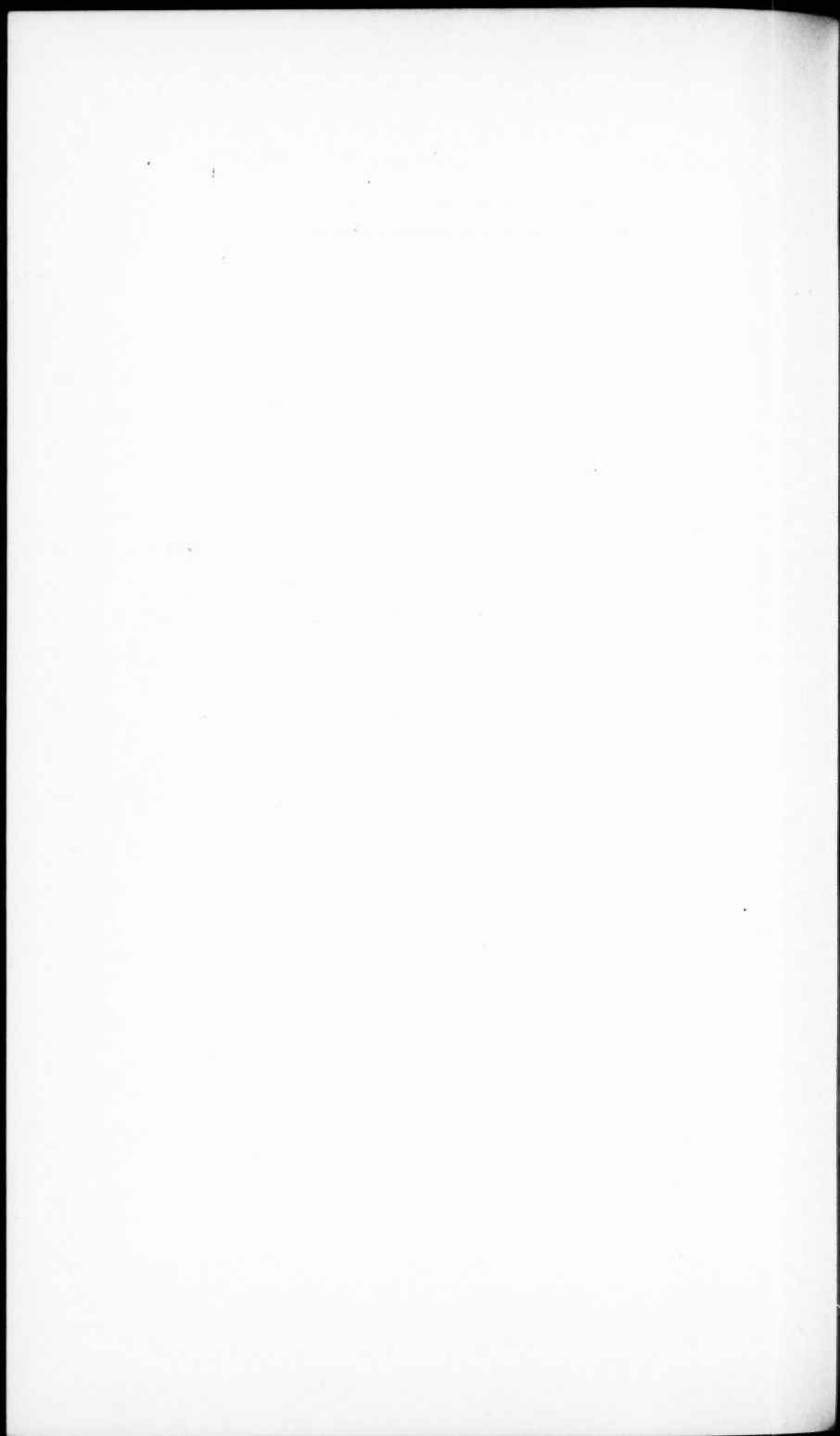
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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

*THE MEASUREMENT OF HYDROSTATIC PRESSURES
UP TO 20,000 KILOGRAMS PER
SQUARE CENTIMETER.*

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IN these Proceedings, Vol. XLV, Numbers 8 and 9, 1909, two methods of measuring hydrostatic pressures were described and applied up to 6800 kgm./cm.². The first method was by means of an absolute gauge of the freely moving piston type; the second method depended ultimately on the first, and utilized the change in the electrical resistance of mercury under pressure. The pressure reached with these two forms of gauge was higher than the highest previous accurately measured pressures, which extended to only 3000 or 4000 kgm./cm.²; but since the earlier paper, the region of attainable and measurable pressures has been still further extended to over 20,000 kgm., so that a re-examination of the gauges there proposed became necessary. Both of the previous methods were found to become inapplicable at pressures much higher than 6800 kgm., the first because of the yielding of the steel of the gauge and the second because of the freezing of the mercury. In this paper the modifications of these two methods are described with which it has been found possible to reach these higher pressures. The freely moving piston gauge has been changed in design so that it has been possible to reach 13,000 kgm., and has been provided with a different reading device. The second method, involving the change in resistance of mercury, has been replaced by another method using the change in resistance of manganin wire. With this an indicated pressure of 20,670 kgm. has been reached. This paper is occupied with a discussion of the calibration, the corrections, and the details of manipulation of these gauges.

THE ABSOLUTE GAUGE.

The novel features of the gauge described in the previous paper which made it possible to more than double the pressure range of
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Amagat were the smallness of the piston exposed to pressure (1/16 inch), and the fact that the pressure cylinder in which this piston moved was exposed to pressure on the external as well as on the internal surface, so that the effect of the increasing pressure was to decrease the internal

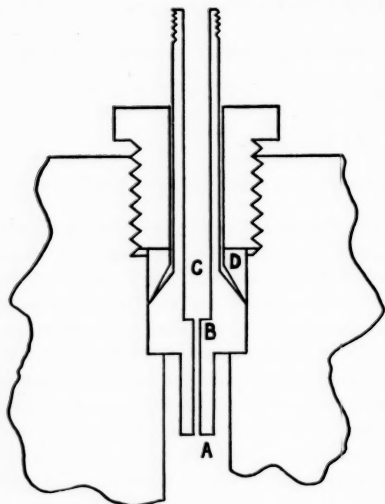


FIGURE 1. The absolute gauge; shows the cylinder of the gauge and the method of mounting it in the containing vessel.

bore of the cylinder and so decrease the leak. The application of pressure to the outside of the cylinder was made through the medium of the packing.¹ The figure referred to shows that the disposition of the packing was such that the cylinder was exposed externally to pressure over the lower end, and to none at all over the upper. At high pressures the effect of such an arrangement is invariably to pinch off the cylinder, the packing forcing its way into the cylinder and separating the upper from the lower end. Two things are necessary to produce this effect: external pressure and non-support of one of the ends. The action is similar to that of a roll of putty which, when squeezed in the

hand, will ooze out sidewise if the ends of the roll are left free, but may be prevented by compressing the ends of the roll between the fingers of the other hand. In the new form of gauge the two novel features, smallness of piston and a cylinder exposed to pressure on the outside, have been retained, but the manner of applying the packing has been so changed as to avoid the pinching-off effect. The new gauge is shown in Figure 1. The cylinder AB is placed lower down than in the former gauge, so that now it is in the part of the metal subjected to hydrostatic pressure only. The packing, which now takes the form of a cone of soft steel, D, is placed above the upper end of the cylinder. This new form

¹ See Figure 2, p. 205, loc. cit.

gains at three points. The soft steel packing, which takes the place of the rubber, does not transmit pressure hydrostatically and so exerts a smaller pinching-off effect; the cylinder is made of hardened nickel steel instead of tool steel, so that it has a higher yield point and more effectively resists what pinching-off effect there is; and the cylinder itself, which is the only vital part, is placed entirely beyond the reach of this effect. Even in this form, however, the soft steel packing does flow sufficiently to produce the beginning of the effect. After the application of the maximum pressure the bore at C showed a very marked decrease. But since nothing depends on the size of the bore at C, no inaccuracy is introduced.

This form gives up one advantage claimed for the former gauge, namely, that by changing the area over which the packing is distributed it becomes possible to some extent to control the distortion of the cylinder and so the leak. But this question of leak proved of much less importance than was anticipated, it being possible to range from 0 to 13,500 kgm. without inconvenient leak at any point. The leak does, however, as anticipated, become less at higher pressures, partly because of closing up of the crack, so that some

sort of control of the leak would become necessary if the crack should ever close up completely. But it has been found possible with the present form of gauge to provide an effective control of leak by providing for the highest pressures a piston slightly smaller than normal. This is evidently easier than to attempt so to design the gauge that the adaptability for different pressures should be secured by changing the packing. The need for even this procedure of changing the piston would probably be slight in practise, for as already stated, one piston sufficed up to 13,500 kgm., and the gauge itself would probably not stand much more.

It is evident that the new form given the gauge makes necessary a recomputation of the correction for distortion. The effect of this distur-

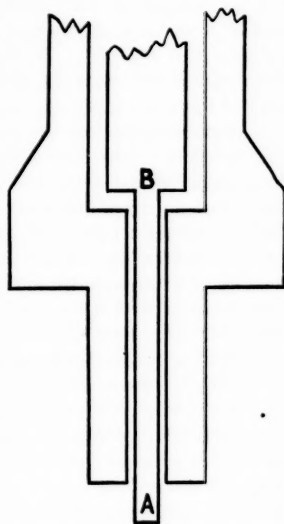


FIGURE 2. The absolute gauge; shows enlarged detail of the cylinder and the piston.

tion is to change the effective area of the piston and so to change the total thrust exerted on the piston by a given hydrostatic pressure. As previously explained, this correction is calculated mathematically and is merely a rough approximation. Its justification is that the correction is in any event exceedingly small, and that no easy experimental method of determining it directly presents itself. The correction is to be made by finding the change in the mean area of the cylinder and the piston at the lower and the upper ends, and taking the mean of these two changes. This gives the change in the effective area of the cross-section as was proved in the former paper.

The stress system on the piston and cylinder is as follows. On the piston there is the longitudinal thrust produced by the action of hydrostatic pressure on the end at A, and the equal and opposite thrust of the equilibrating forces at B. This thrust is uniform throughout the length of the piston. In addition there is the normal pressure on the curved surface exerted by the liquid which is slowly flowing out through the crack between piston and cylinder. At A this pressure is equal to the total hydrostatic pressure, and regularly decreases from here outward to zero at B. On the cylinder there is externally a uniform hydrostatic pressure over nearly the entire length; internally the same distribution of pressure as acts on the piston. At the inner end A the resultant of these two systems of stress is effectively a uniform hydrostatic pressure on both piston and cylinder, under which both shrink uniformly. If r and R are the radii of piston and cylinder initially, and r' and R' the corresponding values under pressure, we have evidently:

$$\text{At the lower end} \quad r' = r \left(1 - \frac{pk}{3} \right)$$

$$R' = R \left(1 - \frac{pk}{3} \right).$$

$$\text{New effective radius} \quad = \frac{r' + R'}{2} = \frac{r + R}{2} \left(1 - \frac{pk}{3} \right).$$

$$\begin{aligned} \text{Effective area changed by} \quad & - \frac{2pk}{3} \\ & = - 4 \times 10^{-7} \times p \\ & \quad \left[\begin{array}{l} p \text{ is pressure in kgm./cm.}^2 \\ k = \text{compressibility} = 6 \times 10^{-7} \end{array} \right] \end{aligned}$$

At the end B, the piston is exposed merely to the thrust p . It will be assumed that the strain is the same as that under a thrust p uniform throughout the entire length.

$$\begin{aligned}\text{Then at B} \quad r' &= r \left(1 + \frac{p\sigma}{E} \right) \\ &= r (1 + 1.4 \times 10^{-7} \times p),\end{aligned}$$

where σ is Poisson's ratio, assumed = 0.28, and E = Young's modulus, taken as 2×10^6 kgm./cm.².

This correction for the distortion of the piston is not open to serious question, because of the smallness of the diameter compared with the length. But the calculation of the distortion of the cylinder at the upper end is open to much more serious question, because the irregular shape and unknown action of the packing produce an unknown stress system in the mass of metal about the upper end. In place, then, of a calculation, the experimental fact was used that in all probability the crack between piston and cylinder would not completely disappear at less than 15,000 kgm., since the piston still possessed some freedom of motion at 13,000. Discussion of this experimental fact is given later. It will be assumed, then, simply that the distortion is proportional to the pressure, and that the combined distortion of cylinder and piston will produce complete closing at 15,000. The initial size of the crack was determined experimentally, by measuring the piston and the effective area, to be 0.00035 inch.

This gives at the upper end $R' = R (1 - ap)$.

When the crack closes $r' = R'$

$$\text{or} \quad r (1 + 1.4 \times 10^{-7} \times p) = (r + 0.00035) (1 - ap).$$

$$\text{Substituting} \quad r = 1/16,$$

$$\text{we find} \quad a = 2.3 \times 10^{-7}.$$

$$\text{At B} \quad R' = R (1 - 2.3 \times 10^{-7} \times p).$$

$$\text{Whence} \quad r' + R' = r + R + \frac{r + R}{2} [(1.4 - 2.3) \times 10^{-7} \times p].$$

$$\text{Effective radius, } \frac{r' + R'}{2} = \frac{r + R}{2} (1 - 0.45 \times 10^{-7} \times p).$$

$$\text{Effective area decreased by } 0.90 \times 10^{-7} \times p.$$

The average change of effective area, top and bottom, which is the correction desired, is therefore $- 2.4 \times 10^{-7} \times p$.

The correction found in this way is to be regarded as an upper limit, the assumption being made that the crack will close up at 15,000 kgm. The correction found by this assumption is somewhat doubtful. A lower limit to the correction can be found by making the assumption as far removed as possible from that made above, namely, that at the upper end the cylinder suffers no change of internal radius. The change of effective area at the upper end is due to enlarging of the piston alone, therefore, and is evidently $1.4 \times 10^{-7} \times p$. In this case the change in the effective area is $1.3 \times 10^{-7} \times p$. The upper and the lower limits differ only 1/10 per cent at 10,000 kgm. The correction applied in the following work was taken as the mean of these two limits, $1.8 \times 10^{-7} \times p$, and the observed gauge readings have been increased in this ratio. This happens to be the same as the correction used for the former gauge.

The experimental evidence used in part of the above approximations requires brief mention. The calculations given above show a more rapid closing of the crack at the end B, so that the tendency of the piston would be to bind at the upper end. This was verified experimentally by the fact that the upper end of the piston was always more brightly polished after a little use than the lower end, and that sticking could be avoided at the higher pressures by making the piston slightly conical. To accomplish this it was sufficient to make the upper end 0.0002 inch less in diameter than the lower. The figure above for the initial width of the crack at the upper end (0.00035 inch) was obtained by combining with the conicality the measured value for the effective area to be described later.

In actual use care was necessary to be sure that the sticking was really due to closing of the crack, and not to viscosity in the fluid transmitting pressure. In the early experiments, in which the transmitting fluid was molasses and glycerine, almost complete sticking was found at pressures as low as 7500. That this was not due to closing of the crack was shown simply by warming the whole apparatus, thus decreasing the viscosity without materially decreasing the size of the crack. It was thus possible to reach 13,000 with very much less sticking than at 7800 at the lower temperature, and also with much less leak, showing an actual decrease in the size of the crack at high pressures. The liquid finally adopted for use in this work was a mixture of glucose with glycerine and water. Glycerine and water were first mixed in equal parts, and then the glucose thinned with this mixture to a best consistency found by experiment. The advantage is that the pressure effect on viscosity is much less than for the molasses and glycerine mixture, so that a mixture of given consistency will work over twice the pressure range of the molasses mixture.

The functioning of the gauge at high pressures is therefore prevented by two effects, — increased viscosity of the liquid, and closing of the crack. In view of the fact that the gauge still worked at 13,500 when both these effects were operative, the estimate made above that the functioning would cease at 15,000 by the closing of the crack only would seem to be amply low. The maximum value set on the correction above is probably, therefore, too high.

The calibration of the piston and cylinder at low pressures to determine the effective cross-section was carried out by the method used in the previous paper. Some such indirect method of calibration was made necessary by the fact that the dimensions of the small piston are so small as to make accurate direct measurements of its effective diameter impossible. The method consisted in hanging weights on the piston to be calibrated and on a larger piston of known area, in such a proportion that the pressure produced by the two pistons should be the same. This is done most simply by connecting the two freely moving pistons to the same pressure chamber, keeping the weights on one piston invariable, and changing those on the other until neither rises or falls. The details of the method were the same as that described before, the same comparison piece of apparatus being used.

The results of the comparison showed an effective diameter for the piston of 0.06250 inches. The measured diameter was only 0.0623 inches at the larger end and 0.0622 inches at the center, showing a crack between piston and cylinder 0.0003 inches wide at the center, 0.00025 inches at the lower end, and 0.00035 inches at the upper end, as used in the calculation above.

In the earlier measurement of high pressures, the thrust was found by hanging weights directly on the piston, and determining by trial that weight which produced neither rise nor fall of the piston. This has the advantage of ideal accuracy, but has several serious disadvantages of manipulation. Flexibility of design in the apparatus is sacrificed because the gauge must be kept vertical. The scale pan and weights become increasingly cumbersome at high pressures, so that an assistant is needed. And worst of all, it requires considerable time to make a reading. This is a fatal objection where the pressure must be read instantaneously, as in experiments to be described in a following paper on the freezing of mercury by the method of electrical resistance.

The present gauge was made direct reading and instantaneous by causing the thrust to produce a measurable deflection in a stiff spring. The new process is related to the old exactly as weighing with a spring balance is to weighing with separate weights. The spring balance is less accurate, but very much more convenient. The accuracy obtain-

able with the new device was, however, sufficient for all requirements. There is an added advantage in that the stroke of the piston over the entire pressure range may be very much decreased. The stroke of 12 mm. in the previous work was reduced to 1.5 mm. in this form. This ensures greater strength at the upper end where the piston pro-

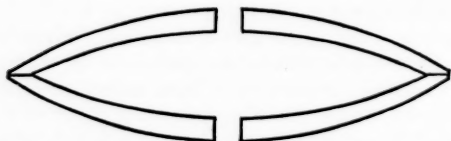


FIGURE 3. The springs with which the thrust on the piston is measured.

jects unsupported from the cylinder, and also ensures greater accuracy, the piston always playing in approximately the same part of the cylinder.

Several forms of spring were tried. The form finally adopted as giving the great stiffness desired without inconvenient bulk was that of a saucer. Two of these saucer-shaped springs were used, placed rim to rim, and the thrust applied at the center (see Figure 3). This arrangement has the advantage of avoiding all friction when the circumference of the saucer increases under pressure, since here the two saucers support each other and move together without slipping.

The choice of steel is of great importance. The most suitable of the many kinds tried was one manufactured by the Halcomb Steel Co. in the form of sheets.

Much experimentation was necessary before the best dimensions and the best heat treatment of the springs was found. The sheet from which the springs were cut was 0.105 inches thick. This was cut into discs 2 1/2 inches in diameter, with a 1/4-inch hole through the center. The disc was then turned on one flat face so as to decrease regularly from 0.105 inches at the center to 0.035 inches at the edge. It was given the saucer shape in a die while hot, the depression at the center being about 1/4 inch. The greatest care was necessary in hardening to prevent warping. This was conveniently done by heating to a bright red heat in a bath of molten lead so as to secure a uniform heat, and then quenching by plunging edgewise into water. The temper was drawn by heating to 370° C. in oil. The temperature was important: 360° C. was too low. Finally, after tempering, the springs were ground flat on the edge to provide a bearing surface against each other. Warping due to hardening was most distinctly shown by varia-

tions in the width of the ground strip. A pair of springs so made will support indefinitely at the center a weight of 1350 pounds without permanent set. The deflection under this load is 2 or 3 mm. The actual working pressure did not exceed 650 lbs.

The small motion of the springs was magnified by a simple mirror device, and observed with a telescope and scale rigidly attached to the frame holding the springs. The scale distance was only 30 cm., but it was nevertheless possible to obtain a magnification of over 1500 times with perfect consistency and freedom from back lash or tremor. The magnification was doubled by reflecting twice from the moving mirror. The size of the piston, sensitiveness of the springs, and optical magnification were altogether such that 8 kgm./cm.² on the gauge produced a deflection of 0.1 mm. at the observing telescope. This gives 1/10 per cent as the accuracy of the pressure readings at 8000 kgm./cm.²; proportionally more at the higher pressures.

All of the parts connecting together the springs and the mirrors were made of steel. This has the advantage of avoiding any motion of the mirrors which might be produced by changes of temperature of the surrounding atmosphere. The only temperature effect is that due to the change in the elastic constants of the steel spring with variations of temperature. This was so small that no appreciable error is introduced under the ordinary conditions of use.

Any device for measuring the magnitude of a stress by the deflection of a spring must be subjected to pretty careful scrutiny before the measurements can be accepted as accurate, because there are disturbing effects, such as elastic after-working and hysteresis, which complicate matters. It was hoped to reduce these effects to a negligible value by using as the working stress less than half the stress at the elastic limit as mentioned above. But even with this precaution it seemed desirable to calibrate carefully the springs under working conditions.

In order to facilitate the comparison, the springs, multiplying mechanism, and telescope and scale were rigidly connected in one piece. This could be screwed either to the end of the absolute gauge for the purpose of measuring the thrust on the piston, or to the calibrating device. The calibration was effected at first by hanging weights on a stirrup, but this process, always discontinuous and sometimes as complicated as applying two, removing one, applying two, etc., was so unlike the process of loading during actual use that another method was seen to be necessary. Two freely moving pistons were used, as when finding the area of the 1/16-inch piston, both communicating with a Cailletet pressure pump of the Société Genevoise. One piston, 1/4 inch in diameter, was the same as that used in the previous work. Weights were

suspended directly from the upper end of this piston. This piston was kept in constant rotation by a small motor so as to avoid friction. The other freely moving piston, $5/16$ inch in diameter, was in direct hydrostatic communication with the $1/4$ -inch piston, and at its upper end pressed directly against the springs to be calibrated. (This $5/16$ -inch piston could be rotated by hand as occasion required to destroy friction.) From the weight on the $1/4$ -inch piston, and the areas of the $1/4$ -inch and the $5/16$ -inch pistons, the thrust on the springs could be calculated. The area of the $5/16$ -inch piston was found in the same way as that of the $1/16$ -inch piston. The method has all the advantages of the discarded method of the direct application of weights, namely, complete freedom from all elastic effects and hysteresis, and in addition permits very much more convenient and flexible application of pressure.

The procedure of the calibration was to place a weight on the $1/4$ -inch piston, completely depressing it. The piston was then floated again by raising the pressure to the equilibrium value with the Cailletet pump. The $5/16$ -inch piston was then rotated to destroy friction, and the deflection of the spring read. Eleven such steps were made with increasing and decreasing pressure, making twenty-two steps in all. The same weights were used in all the calibrations, so that the results were strictly comparable. The pressure exerted by the fluid, as given by the gauge of the Geneva pump, was also recorded as a check. The accuracy of the other readings was so great, however, that these check readings could never be used.

Calibration with this device was first made to find whether the gauge had any error of position, since it was generally calibrated vertically, but used horizontally. This could evidently be done very simply by changing the position of the cylinder with the $5/16$ -inch piston, a change in the calibrating procedure which could not be made so simply when weights were directly applied. The result of the calibration in the horizontal position showed no detectable error due to this change of position. Of course no such error was to be expected, since all the parts were very stiff in comparison with their weights.

A second result of the calibration was that the springs show no elastic after-effects. By this is meant the gradual creep after application of a load, and gradual recovery after removal. The effect is generally most pronounced at the two ends of the pressure range. These springs, however, showed no tendency to yield viscously under the maximum stress, and never showed any wandering of the zero after release of pressure.

A third result of the calibration was that the springs do not follow

the linear law, but are increasingly deflected at the higher pressures. The change was not large, but perfectly distinct, about 6 per cent less weight being required to produce a given deflection at 650 lbs. than initially. This is rather surprising in a substance like this spring steel, which ordinarily follows the linear law to the elastic limit. The

TABLE I.

RELATION BETWEEN LOAD AND DEFLECTION OF SPRINGS WITHOUT DEVICE FOR AVOIDING HYSTERESIS.

Load Kgm.	Mirror Deflection.		Load Kgm.	Mirror Deflection.	
	Increasing Load.	Decreasing Load.		Increasing Load.	Decreasing Load.
00.00	0.00	0.00	95.62	8.52	8.64
16.26	1.42	1.49	112.27	10.06	10.18
33.37	2.94	3.02	128.27	11.55	11.66
49.35	4.35	4.45	143.32	12.97	13.07
65.54	5.81	5.91	160.18	14.57	14.64
81.59	7.24	7.36	173.85	15.90	

effect is evidently due here to the change of geometrical shape, the springs becoming so much flatter under the higher stresses that the geometrical configuration as such has a lower elastic constant, although the elastic constant of the material itself is unaltered. It is customary in the mathematical treatment of the bending of thin rods or plates to assume that the deflection remains proportional to the stress up to the elastic limit. This experiment shows that this approximation may become invalid at considerably less than the elastic limit.

A fourth result was that the gauge does show some hysteresis, a result which was not expected in view of the second result. For as a general rule, hysteresis and elastic after-effects, while not directly related, occur together, both being evidence of some molecular instability. Table I., giving the difference between the reading under increasing and decreasing pressure, shows the usual magnitude of the effect. The first column in the table gives the total load at each step. The effect is much less than that due to departure from linearity mentioned above, so that here we have a hysteresis loop of the unusual shape shown in Figure 4. The lag in similar cases is

usually so great that the curvature with decreasing stress is the reverse of that with increasing stress, so that the loop has the general shape of a double convex lens. Furthermore the loop is usually very nearly symmetrical with respect to the line joining the extremities. This is the only example of a loop of the above shape known to the

writer. This single example is sufficient to show that there is no necessary connection between hysteresis and departure from the linear relation between stress and strain, as might be supposed if all loops were of the ordinary type.

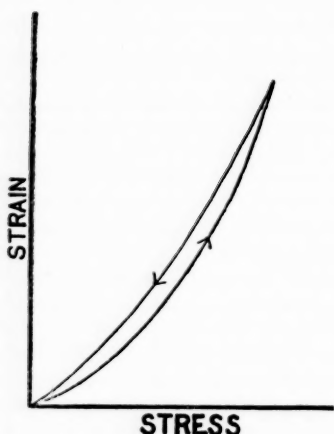


FIGURE 4. Shows the unusual nature of the hysteresis cycles described by the springs.

This hysteresis, while comparatively small, was nevertheless sufficient to reduce the accuracy of measurements made with the gauge far below that desired. A method of avoiding hysteresis was therefore adopted. It depends on the fact, well known for magnetism, that if the stress is varied cyclically by small amounts about any fixed point, a small hysteresis loop is described about this point. The result is that if stress is relieved from the point A (see

Figure 5) after increasing pressure, the path AB will be described, while if it is increased from the point C after decreasing pressure, the path CB is described. Suppose that during increasing pressure the point D has been reached, or that during decreasing pressure the same stress, shown at E, has been reached. The difference between the points E and D represents the error due to hysteresis. To make the readings at these two points the same, we may evidently apply a small extra load at D, raising the stress to A, and then remove the extra load, or at E we may remove a slight portion of the load to C and then reapply it. The same point B is finally reached, and the pressure readings have become single valued. The extra load necessary to apply or remove must be determined by experiment, and would be expected to vary at different parts of the hysteresis loop.

This extra load was applied in practise by a very simple lever arrangement, by which the springs could be deflected one way or the

other before making readings. The following set of readings, picked at random from a great number, shows how nearly it was possible to avoid hysteresis by the method. The difference of readings is seldom more than the possible error of reading. The small extra load applied or removed before making these readings was constant over the entire range, being sufficient to produce a deflection of about 2.0 divisions. It is evidently not quite enough at the higher pressures and a little too much at the lower pressures. The mean of the two readings nowhere differs from either reading by more than the errors of observation, however, and this simpler procedure was therefore adopted.

The most inconvenient fact disclosed by the calibration was that the constant of the springs varies slowly from time to time. Over two or three days no change whatever is to be noticed, but in a week or a month there are likely to be changes beyond the limits of error. The change is irregular and has no apparent connection with temperature changes. No temperature effect within the limits of working room temperatures was ever found. The change is doubtless due to some slow process of molecular accommodation going on within the metal itself. At one time the springs were permanently deformed by a violent explosion, so that the deflection under the same load was increased in the ratio 16:14. After this deformation for a month or more the change with time was more rapid than usual. The table shows the magnitude of the variation with time. The first set of three, January–April, 1910, was made at intervals during constant use of the gauge. During this time the gauge constant decreased and then increased again. The explosion referred to above took place on December 24, 1909. The last two readings, September 6 and 24, 1910, were made after the springs had been resting for about four months. The constant is in general higher, but the greatest increase has come at the middle of the range, so that the relation between stress and strain is more nearly linear than before. After this prolonged period of rest, the gauge has remained much more nearly constant than before.

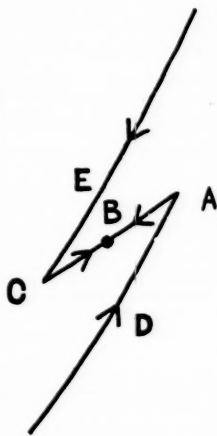


FIGURE 5. Detail of a hysteresis loop, showing the method of avoiding the effects of hysteresis in making the readings with the springs.

TABLE II.

RELATION BETWEEN LOAD AND DEFLECTION OF SPRINGS WITH DEVICE FOR AVOIDING HYSTERESIS.

Load Kgm.	Mirror Deflection.		Load Kgm.	Mirror Deflection.	
	Increasing Load.	Decreasing Load.		Increasing Load.	Decreasing Load.
00.00	0.00	0.00	95.62	8.50	8.50
16.26	1.43	1.42	112.27	10.02	10.04
33.37	2.94	2.92	128.27	11.51	11.54
49.35	4.35	4.34	143.32	12.98	12.94
65.54	5.80	5.79	160.18	14.54	14.54
81.59	7.23	7.23	173.85	15.85	

TABLE III.

SHOWING VARIATION OF SPRINGS WITH TIME.

Load Kgm.	Mirror Deflection.				
	Jan. 1.	March 30.	April 27.	Sept. 6.	Sept. 21.
16.26	1.65	1.63	1.65	1.66	1.66
33.37	3.37	3.32	3.36	3.38	3.38
49.35	5.00	4.95	4.98	5.02	5.03
65.54	6.68	6.58	6.65	6.70	6.70
81.59	8.37	8.25	8.35	8.39	8.40
95.62	9.86	9.74	9.83	9.88	9.89
112.27	11.66	11.63	11.64	11.69	11.70
128.27	13.42	13.28	13.40	13.44	13.47
143.32	15.09	14.96	15.09	15.14	15.15
160.18	17.00	16.88	17.02	17.05	17.08
173.85	18.60	18.52	18.63	18.63	18.65

This change of gauge constant with time demands that frequent calibration be made. In all the work of the following papers in which considerable accuracy was desired, such calibrations were made every few days. It is not much trouble to make this calibration. The twenty-two readings with increasing and decreasing pressures can be made in the course of half an hour. When the gauge is so calibrated, and the procedure for avoiding hysteresis is adopted, the readings are consistent to the limit of sensitiveness. This has already been stated to be about 8 kgm./cm.²

The gauge in actual use has shown itself very convenient. The readings may be made rapidly and immediately after the application of pressure, since there are no thermal effects such as dissipation of heat of compression. The temperature effect is so small that it may be used directly in the room without a thermostat, and the leak is so slight that it may be used in entire comfort in measuring many high-pressure effects. Where applicable, the gauge is more convenient than the electrical resistance gauge and it has been used whenever possible.

THE MANGANIN RESISTANCE GAUGE.

In the earlier paper a method was described for measuring high pressures by measuring the electrical resistance of pure mercury under pressure. The method had the advantage of being perfectly reproducible, so that any one could at any time measure pressure without recourse to the then inconvenient fundamental standard of pressure. The advantage has been in large part offset by the devising of the convenient form of absolute gauge described in the first part of this paper. Moreover, the method becomes inapplicable at somewhat higher pressures than those reached formerly because of the freezing of the mercury. Thus at 0°, the freezing pressure is about 7500 kgm./cm.². Aside from this difficulty, which might be avoided by placing the mercury in a separate vessel, maintained at higher temperature, there are numerous inconveniences of manipulation, as, for instance, that the mercury must always be kept in an upright position. But the greatest inconvenience of all is that the glass capillary containing the mercury is always shattered by an explosion, and explosions become more and more frequent at high pressures.

There are frequently situations, however, where the absolute gauge becomes unavailable, and where a gauge with some of the properties of the mercury gauge becomes desirable. For instance, it is often necessary to secure absolute freedom from leak, and this is obviously impossible with a freely moving piston as in the absolute gauge.

Measurements of compressibility or of change of volume during freezing, as in two following papers, demand this property of freedom from leak.

The method of pressure measurement adopted here, and which secures freedom from leak, has already been described by Lisell.² He measured the effect of pressure on the resistance of manganin wire, and proposed that the change of resistance be used as a measure of pressure. Lisell found the effect of pressure between 0 and 4200 atmos to be linear, and showed that there was no appreciable temperature effect between 0° and room temperature. But Lisell also showed that different specimens of manganin show slightly different pressure coefficients, so that the advantage of reproducibility must be given up. Lately, Lafay³ has also measured the pressure effect on manganin up to 3500 kgm. and has found nearly the same pressure coefficient as did Lisell.

The data of this paper show by direct comparison with the absolute gauge that the manganin is suitable as a pressure gauge over much wider ranges of pressure. Up to 13,000 kgm. the relation is linear within the errors of the absolute gauge. It would not have been surprising if this had not been true, in view of the fact that manganin has a pressure coefficient which is positive instead of negative like that of all the pure metals. Furthermore, over this pressure range the readings are entirely free from hysteresis or creep. This is opposed to the work of Lussana,⁴ who found various temporary effects after the application of pressure. His results have not been verified by subsequent observers, however, and the entire absence of the effect here under a very much wider pressure range would seem to make pretty certain that there was some obscure source of error in Lussana's work.

The manganin used in this work was of German manufacture, No. 38, double silk covered, of about 30 ohms to the meter. With change of temperature it shows a maximum resistance at about 27°. The coils were of about 100 ohms resistance, the wire being wound non-inductively on itself in the form of a toroid, about 1 cm. in diameter and 5 mm. thick. To protect the wire, the toroid was covered with a winding of fine silk ribbon, only the ends of the wire being exposed in order to make connections with the insulating plug. This plug was of the same design as that shown in the previous paper, except that it was made of hardened nickel steel instead of tool steel.

² Lisell, *Om Tryckets Inflytande på det Elektriska Ledningsmotståndet hos Metaller samt en ny Metod att Mäta Höga Tryck* (Diss. Upsala, 1903)

³ Lafay, *C. R.*, **149**, 566-569 (1909).

⁴ Lussana, *Nuov. Cim.*, **10**, 73-84 (1899); **5**, 305-314 (1903).

The apparatus for producing and measuring pressure consisted of two cylinders connected by a tube of nickel steel. In the lower cylinder the pressure was produced and measured. The absolute gauge of the first part of this paper was screwed directly into the side of this cylinder. The cylinder itself was of Krupp chrome nickel steel, 8 inches outside diameter, 1 1/8 inches inside diameter. It was placed in a hydraulic press of 200 tons capacity, and the pressure produced by a 1 1/8-inch piston forced into the cylinder by the ram of the press. The upper cylinder, also of Krupp chrome nickel steel, 4 1/2 inches outside and 9/16 inch inside diameter, contained the manganin wire to be tested. The connecting tube to the lower cylinder passed through the bottom of the tank of a thermostat, which surrounded the upper cylinder, and with which the temperature could be kept constant to 0.01°. No such temperature precaution was necessary for the absolute gauge. The lower cylinder was filled with the mixture of glucose and glycerine needed to secure tightness of the piston of the absolute gauge, and the upper cylinder was filled initially with either kerosene or gasolene, in which the manganin coil was directly immersed. The action of pressure was to compress the kerosene, glucose passing from the lower cylinder to the lower part of the upper cylinder. The compression was never sufficient, however, to bring the glucose into contact with the manganin. Although kerosene or gasolene are somewhat inconvenient because of their high compressibility, still their use was made necessary by the fact that a heavier oil freezes under pressure, so that it does not transmit pressure hydrostatically to all parts of the wire. The kerosene is also known to become stiff like vaseline at say 10° and 8000 kgm., but the viscosity is not so great as to introduce irregularities. When either kerosene or gasolene is used, the insulating qualities of the plug are practically perfect without requiring any special precautions. The insulation resistance was always over 10 megohms, which was the limit of the measuring device conveniently at hand. Formerly, in working with mercury, pressure was transmitted by a mixture of water and glycerine. It was necessary to specially protect the separate parts, and even then the insulation resistance was never greater than several hundred thousand ohms.

The electrical measurements were made by the same null method and on the same Carey Foster bridge as those described in the former paper.

In making the calibration and in using the gauge, there is one fact to be borne in mind which has its analogy in the mercury resistance. This is the seasoning effect of pressure; the gauge does not respond to the first application of pressure in the same way that it does to the

second or subsequent applications; there is a gradual settling down to a steady state. In the case of mercury, this was due to the equalization of strains in the containing capillary of glass. In the case of the manganin, some such process of accommodation must be going on within the mass of the metal. This is shown principally by drift of the

TABLE IV.

PRESSURE COEFFICIENT OF RESISTANCE OF MANGANIN.

Pressure Kgm./cm. ² .	$\frac{\Delta R}{pR_0} \times 10^6$.	Pressure Kgm./cm. ² .	$\frac{\Delta R}{pR_0} \times 10^6$.
1260	2272	9290	2302
2610	2282	8000	2304
3810	2299	6450	2302
5000	2301	4790	2304
6180	2299	3250	2291
7210	2300	1740	2308
8230	2302	1000	2272

zero, but the pressure coefficient may change slightly. The seasoning process occupies more time for the manganin than for the glass; it may extend over as much as a month after the first application. It is hastened by frequent applications of pressure, but may apparently run to completion in sufficient time after only one application. To effect the seasoning, it does not seem to be necessary to subject the coil to the maximum pressure under which it is contemplated using it. The coil used to the highest pressure reached in this work, 20,500 kgm., had been seasoned by the application of not more than 12,000 kgm., yet it showed no further change after the application of a pressure 8500 kgm. in excess of the seasoning pressure.

The results obtained with one such well-seasoned coil at 0° are shown in the table. In making the comparison with the absolute gauge all the precautions described in the first part of the paper were observed. It is seen that within the limits of error of the pressure readings, the change of resistance is proportional to pressure.

To show within what limits different pieces from the same spool of wire give the same results, three coils were made from the ends and the middle of a length of wire of 70 m. The constants for the separate coils at 0 were .052301, .052307, and .052325, in the order of the coils, a variation of one per cent. The temperature effect was found by measuring these same coils again at 50°. These same three coils gave 2295, 2319, and 2320 respectively. One of the coils measured at -12° showed no measurable difference between -12° and 0°.

The maximum pressure to which these calibrations were made varied somewhat with the temperature, because at the lower temperatures the mixture of glucose and glycerine used with the absolute gauge became viscous so rapidly with increasing pressure as to transmit pressure very slowly. At the low temperatures the pressure was increased until this limit was reached. The slow flow of glucose from the lower to the upper cylinder might occupy an hour or more before the equilibrium was complete. The fact that there was such a process of flow was definitely shown by the slow fall of pressure in the lower cylinder as indicated by the absolute gauge, with a simultaneous slow rise of pressure in the upper cylinder, as indicated by the manganin resistance. At 0° the maximum reached was in one case 11,000 kgm. This was probably too high for complete equalization of pressure, for the change of resistance was 1/2 per cent too low at this maximum. The measurements at 0° were not usually carried as far as this, 9500 being the more usual limit. Slight differences in the composition of the glucose mixture made very pronounced differences in the viscosity at high pressure. At 50° the highest reached was 12,000. Even here the viscosity was very considerable. On one occasion pressure was pushed to 13,000 at 50°. There was the same slow equalization of pressure, extending over about half an hour, as was found at 0°. At the end of this time the resistance had nearly acquired the value given by a linear relation, when the experiment was terminated by an explosion. The final reading below this at 11,500, at which there was also some viscous yield, completely satisfied the linear relation.

We may conclude, therefore, that over the temperature range 0°-50° the pressure resistance relation is linear within 1/10 per cent of the change of resistance, up to 13,000 kgm. This was proved by actual experiment at 50° to 12,000, and to 9500 at 0°. The extrapolation to 13,000 at 0° is comparatively slight, and is made all the more probable by the fact that our usual experience would lead us to expect greater departure from linearity at higher temperatures, and no such departure was found.

Although different specimens of manganin do not have the same constant, still it may be worth while comparing the results found here with those of other observers so as to give an idea of the magnitude of the variation of the effect. Lafay⁵ found 2.16×10^{-6} per kgm./cm.² as the effect on one specimen. Lisell⁶ found for two specimens of annealed wire 2.13×10^{-6} and 2.08×10^{-6} ; for three specimens of hard drawn wire results from 2.279×10^{-6} to 2.338×10^{-6} . The data of this paper, which are for hard drawn wire, vary from 2.295×10^{-6} to 2.325×10^{-6} .

Four of these manganin resistance gauges have been in almost constant use for over a year. As compared with the mercury gauge they have had the advantage of greater convenience and ease of manipulation, even over the pressure range within which the mercury remains fluid. During the work explosions have been of frequent occurrence, the shock of any one of which would have broken the glass capillary containing the mercury. It is not necessary to apply elaborate temperature precautions as for the mercury. The only temperature correction necessary to apply is a small one for the shift of the zero. This may be determined accurately enough by hanging a thermometer in the air of the room near the cylinder containing the manganin. At room temperatures of 20° a change of temperature of 1° demands a pressure correction of only 5 kgm.

As compared with the absolute gauge of the first part of the paper, each form has distinct advantages. Where available, the absolute gauge is more convenient, because it is direct reading and immediate. But the absolute gauge has the disadvantage of leak, so that often the manganin gauge becomes absolutely necessary. Furthermore, it has the disadvantage of being more cumbersome. This means that all parts of the apparatus in connection with the gauge must be correspondingly enlarged. This is often a fatal disadvantage, entirely apart from any considerations of expense or convenience, because in order to reach the highest pressures, the steel parts must be hardened, and it is not possible to harden large steel cylinders. The upper limit of pressure attainable will have to be reached with comparatively small apparatus. The reason for not pushing the absolute gauge to its limit was not so much fear of destroying the gauge as the fact that the large 8-inch steel cylinder containing the gauge was of soft nickel steel, and that the yield point would have been reached at 15,000 kgm. This cylinder had been previously seasoned by applying pressures up to 28,000 kgm., which had the effect of increasing the internal diameter

⁵ Lafay, loc. cit.

⁶ Lisell, loc. cit.

from $5/8$ inch to $1\ 1/8$ inches, but even this was not sufficient to raise the limit permanently to over 15,000 kgm.

The compactness of the manganin gauge makes it particularly adapted for working with the highest pressures where everything must be in one piece because of the impossibility of making connecting tubes. The gauge has been so used in a number of experiments on the freezing of water under pressure. The gauge was screwed into one end of a large steel cylinder, the plunger was pushed in from the other end, and the water was in the space between. The dimensions were kept down so that the entire block, together with a part of the hydraulic press, could be placed in a thermostat.

The manganin gauge may be used by extrapolation to measure pressures beyond the reach of the absolute gauge. It has been so used in investigating the freezing of water up to an indicated pressure of 20,500 kgm., and this limit could without doubt be exceeded. The limit is not in the manganin itself, but in the hardened steel parts, which have a tendency to stretch too much at pressures as high or higher than 20,000 kgm. Of course the use of any standard by extrapolation is undesirable, but at present any means of measuring these very high pressures with probable accuracy is welcome. In any event, the extrapolation from 12,000 to 20,000 is very much less than the extrapolation from the previous maximum of 4000 to 12,000, which is here shown by actual experiment to be justified. What is more, it will be an easy matter to translate high-pressure readings in terms of a manganin gauge into absolute pressures, if at any time the direct calibration is extended from 13,000 to 20,000, and proves that a linear extrapolation is not sufficiently accurate.

The only serious disadvantage in the manganin gauge as thus far described, when compared with either the mercury gauge or the absolute gauge, is the fact that it is not readily reproducible, so that each new coil of wire must be calibrated against an absolute gauge. This disadvantage may be obviated by the use of fixed pressures of reference analogous to the melting points of the metals used as points of reference in thermometry. The linearity of the relation between resistance and pressure having been established by the work of this paper, it is necessary to know for each coil only the change of resistance corresponding to a single known pressure in order to fix completely the behaviour of the coil.

Such pressures of reference are given very conveniently by the points of transition between the various kinds of ice and water. For low pressures such a pressure of reference is given by the pressure of transi-

tion from ice I to ice III, using Tammann's ⁷ notation. This pressure is very nearly independent of temperature between -22° and -30° . This particular transformation has the advantage that the reaction runs with very great velocity and is accompanied by a comparatively enormous change of volume, 20 per cent, so that in any piece of apparatus, once given the two phases, the equilibrium pressure is automatically set up almost immediately. This pressure has been found to be 2120 kgm. at -23° . For higher pressures, the transition point at 0° from water to a hitherto unknown variety of ice, ice VI may be used. This point has been found by experiment to be 6370 kgm. Equilibrium is reached more slowly than for the transition I-III, but for work at higher pressures the use of this transition point will give a more accurate calibration. This method of calibration has actually been used, with satisfactory results, for two other coils than the four mentioned above.

The calibration of the manganin resistance without the use of an absolute gauge may of course be effected by comparison with any other accurately measured pressure phenomenon. For instance, the calibration may be made by comparing the manganin resistance with the mercury resistance, the data for which have been already published.

SUMMARY OF RESULTS.

Two gauges for high pressures are described in this paper. The first is an absolute gauge which has been used up to 13,000 kgm. The construction of the gauge is described, the correction for distortion is determined, and the reading mechanism is discussed. A procedure is given for freeing the deflections of the springs with which the thrust is measured from hysteresis. All of the factors may be determined with sufficient accuracy so that the gauge is accurate to the limit of accuracy of reading, 1/10 per cent at 8000 kgm./cm.². The second gauge is a manganin resistance. This is shown to be suitable for the purpose, since there is complete freedom from hysteresis and elastic after-effects. The relation between pressure and resistance is shown to be linear up to 12,000 kgm., but the gauge has been used by extrapolation up to 20,500. The accuracy of the readings with this is at least as great as with the absolute gauge. The relative advantages for various kinds of experiment of these two forms of gauge are discussed. Finally, by the use of standard pressures of reference it is

⁷ Tammann, *Kristallisieren und Schmelzen*, pp. 315-344 (Barth, Leipzig, 1903).

shown that a manganin resistance gauge may be calibrated without direct reference to an absolute gauge.

This investigation was a necessary preliminary to the measurement of various thermal properties of mercury and water under pressure, the expenses of which have been partially defrayed by several liberal appropriations from the Rumford Fund of the American Academy of Arts and Sciences.

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